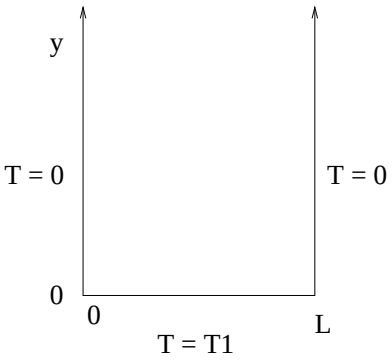
ChE 272 Introduction to Chemical Engineering Calculations University of Rhode Island
Professor Greenfield Spring 2007

Homework 8, due April 5, 2007

Please feel free to use a spreadsheet, Matlab, etc. Late homeworks will be penalized by the complementary error function erfc(x) = 1-erf(x) (see definitions below), with x equal to days / 15.

1. Say you have a thin plate in which the temperature along one edge (width over $0 \le x \le L$) is held constant at T_1 and the temparture along two long edges ($y \ge 0$) equals 0, as sketched in the figure.
   (Each of these temperatures are relative to room temperature.)



   The temperature distribution can be calculated by

$$T = \frac{4T_1}{\pi} \sum_{n=0}^{\infty} \frac{1}{2n+1} \exp\Big(-(2n+1)\pi y/L\Big) \sin\Big((2n+1)\pi x/L\Big)$$

   In this problem we will look at **Fourier series** by seeing how different terms in an infinite series can affect the total. In particular, look at two different y values: $y = L/3$ and $y = L$. For each y value, make a plot of T vs x that compares three different situations. In the first curve, use only the $n = 0$ term in equation 1. In the second and third curves, include terms up to $n = 1$ and $n = 2$, respectively. Next, repeat this process for the value $x = L/2$, making a plot of T vs y, for y between 0 and $3L$.

   How do the results change as more terms are added? What physical situation can you think of that corresponds to this geometry?

2. Chapra and Canale, problem 13.9, part (a) only

3. Chapra and Canale, problem 14.3, part (a) only

4. The *error function* erf(x) arises often in many fields of engineering, because the integrand is a scaled and recentered form of the Gaussian distribution (the famed "bell-shaped curve"). A limit of $y = 1$ is reached in the limit of $x \to \infty$ (by a subtle analytic solution), but for finite x the integral must be calculated numerically.

$$y = \mathrm{erf}(x) = \frac{2}{\pi} \int_0^x e^{-t^2} dt$$

   (a) Estimate the value of $\mathrm{erf}(1)$ using the trapezoidal rule with 10 intervals (11 points).

   (b) Estimate the value of $\mathrm{erf}(1)$ using Simpson's 1/3 rule, with the same 11 points.