Homework 4
due TUESDAY April 29, 2008 (last day of URI classes)

If you find yourself without enough information, make and justify a reasonable approximation.

1. Textbook, chapter seven, problem six (analyze rheology data to obtain viscosity).
2. Textbook, chapter seven, problem sev en (prob six continued, for a power-law fluid).
3. Textbook, chapter seven, problem eigh teen (extrusion)
4. Textbook, chapter seven, problem twen tys ix (injection molding)
5. (*Boltzmann superposition problem.*) An 4mm-thick LDPE sample is created by injection molding. The melt is injected at a temperature of 170°C and a high pressure, with mold walls held at 25°C.

   The objective of this problem is to use the Boltzmann superposition principle (eq 4.83),

$$\sigma(t) = \int_0^t E(t - t')\dot{\epsilon}(t')dt' \tag{1}$$

   to calculate the residual stress $\sigma$ at multiple depths in an injection-molded LDPE sample of thickness $2L = 4$ mm. The temperature at each depth $z$ from the surface decreases with time $t$ as (eq 7.23)

$$\frac{T - T_e}{T_i - T_e} = \frac{4}{\pi} \exp\left(-\frac{\pi^2}{4}\frac{\alpha t}{L^2}\right) \sin\left(\frac{\pi}{2}\frac{z}{L}\right) + \frac{4}{3\pi} \exp\left(-\frac{9\pi^2}{4}\frac{\alpha t}{L^2}\right) \sin\left(\frac{3\pi}{2}\frac{z}{L}\right) + \cdots \tag{2}$$

   where $T_i$ and $T_e$ are the initial (i.e. injection) and equilibrium (i.e. wall) temperatures, respectively, and $\alpha$ is the thermal diffusivity; for this problem use the value $\alpha = 10^{-7}$ m$^2$/s. The surfaces are at $z = 0$ and $z = 2L = 4$ mm. The second term and beyond (see eq 7.23) are often negligible compared to the first after long enough times (see text).

   Assume for this problem that the pressure–volume–temperature relationship leads to a counterbalance between cooling-based contraction and pressure-decrease-based expansion such that local strain initiates (and thermal-based stresses begin to accumulate) in the sample starting at the time when the local temperature reaches 120°C. From that time on, the tensile strain grows as

$$\dot{\epsilon} = \frac{1}{L_0}\frac{dL}{dt} = (COTE)\frac{dT}{dt} \tag{3}$$

   where COTE is the coefficient of (linear) thermal expansion, which equals $2 \times 10^{-4}\text{K}^{-1}$. (See p. 56 for volume expansion and divide by 3; note that COTE is often abbreviated as $\alpha$, which would be confusing here.) $L_0$ equals the length at 120°C. Note that eqs 2 and 3 allow the strain rate and cooling rate to be linked.

The shear relaxation modulus $G(t)$ of LDPE at $T_0 = 150°C = 423.15$ K has been modeled (Laun, 1978, *Rheol. Acta*) using a multicomponent Maxwell model as

$$G(t) = \sum_n G_n \exp(-t/\tau_n) \tag{4}$$

with the following parameter values:

| $n$ | $G_n$ (Pa) | $\tau_n$ (s) |
|---|---|---|
| 1 | 1.00 | $10^3$ |
| 2 | $1.80 \times 10^2$ | $10^2$ |
| 3 | $1.89 \times 10^3$ | $10^1$ |
| 4 | $9.80 \times 10^3$ | $10^0$ |
| 5 | $2.67 \times 10^4$ | $10^{-1}$ |
| 6 | $5.86 \times 10^4$ | $10^{-2}$ |
| 7 | $9.48 \times 10^4$ | $10^{-3}$ |
| 8 | $1.29 \times 10^5$ | $10^{-4}$ |

At temperatures other than 150°C, the time is estimated using time–temperature superposition and the Arrhenius equation,

$$a_T(T) = \frac{t(T)}{t_{\text{ref}}(T_0)} = \exp\left[\frac{E_0}{R}\left(\frac{1}{T} - \frac{1}{T_0}\right)\right] \tag{5}$$

with activation energy $E_0 = 54$ kJ/mol (note that $T$ and $T_0$ are in Kelvin). In other words, the way to use eq 4 at temperatures other than 150°C is to substitute $t/a_T$ for $t$ on the right-hand side.

**On to the specific question!** Use Boltzmann superposition (eq 1) to calculate the residual stress $\sigma$ for the following three times and positions:

(a) $z = 1$ mm from the surface, at the time $t$ that its temperature equals 50°C,

(b) $z = 2$ mm from the surface, at the time $t$ that its temperature equals 50°C,

(c) $z = 1$ mm from the surface, at the time $t$ that the temperature of $z = 2$ equals 50°C.

Numerical integration will likely be required. If so, please use a computer program of some sort. (Feel free to stop by for assistance.)